ITCS
INNOVATIONS IN THEORETICAL COMPUTER SCIENCE

# MPC for MPC: Secure Computation on a Massively Parallel Computing Architecture

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# Models of Parallel Computation

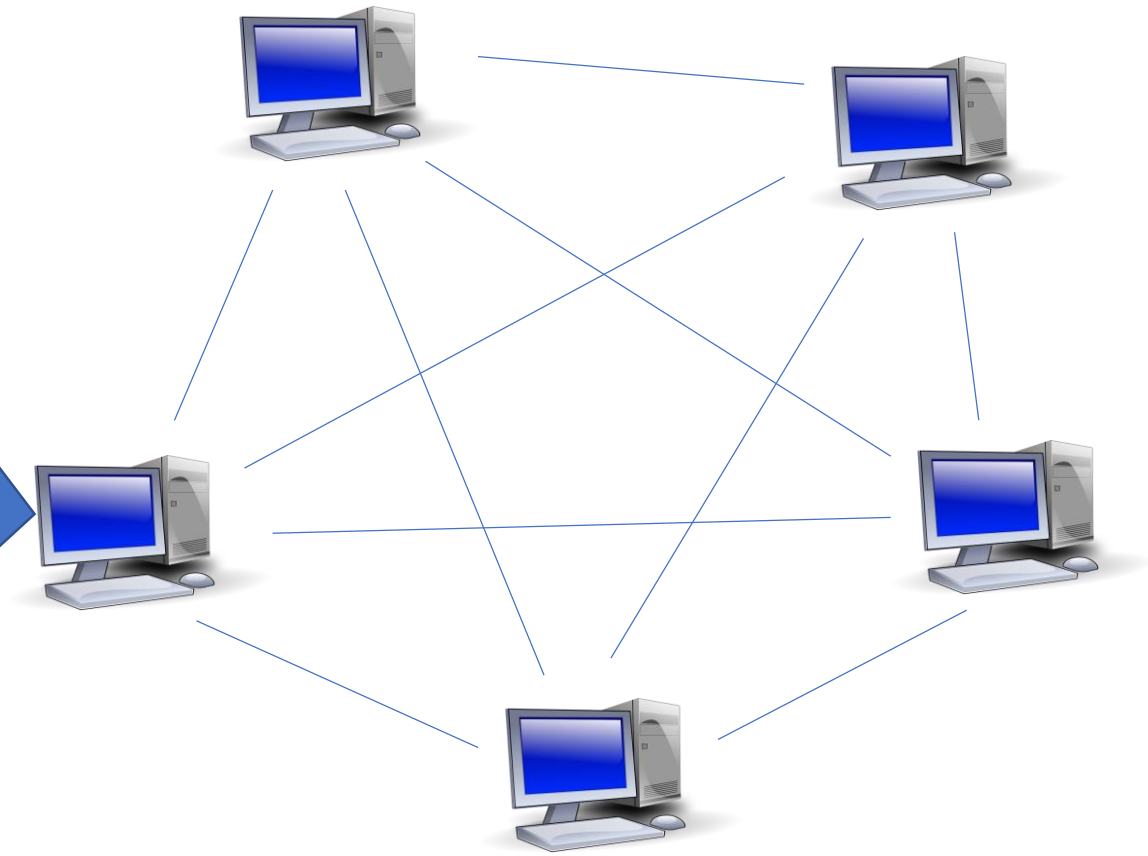
- Circuit?
- Parallel Random Access Machine (PRAM)
- Bulk Synchronous Parallel (BSP) model

Karloff, Suri, and Vassilvitskii (SODA 2010)

**Massively Parallel Computation, MPC**

# Massively Parallel Computation (MPC)

- $m$ Random Access Machines (RAM)
- Fully connected
- Each of space $s$



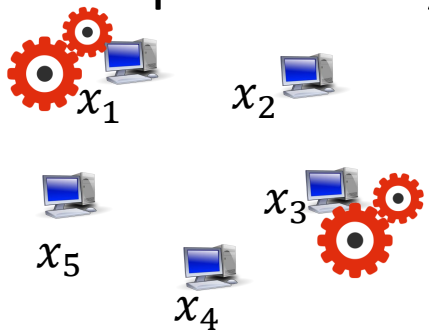
- Input size $N$
- $s = N^{\epsilon}$, const $\epsilon \in (0,1)$
- $\Rightarrow m \geq N^{1-\epsilon}$

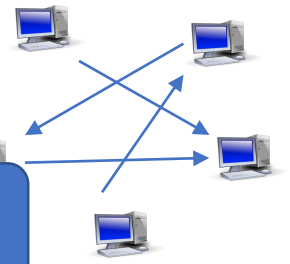
# MPC Proceeds in Rounds

Space $s = N^{\epsilon}$

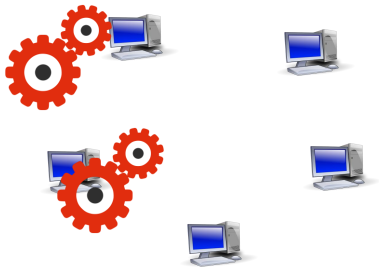
Compute locally

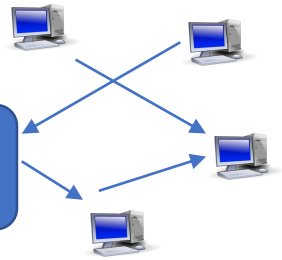
Send messages (send/receive $s$ bits each)



Round 1





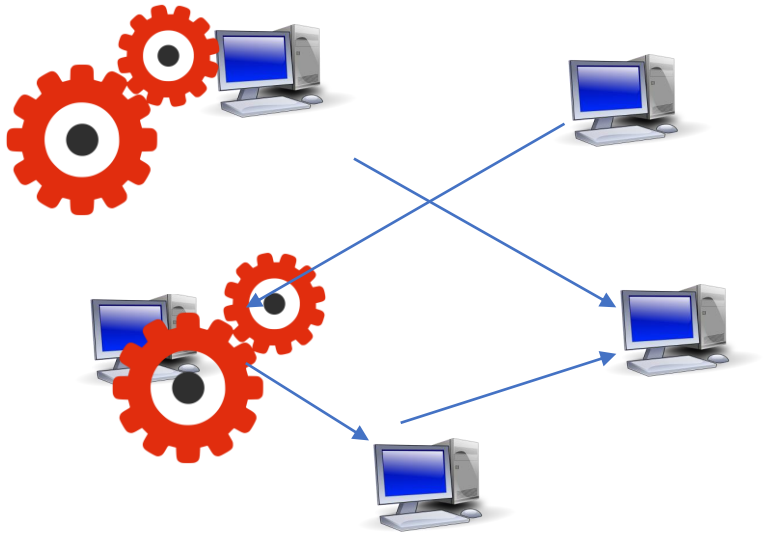
Round 2

Repeat rounds .... Output jointly

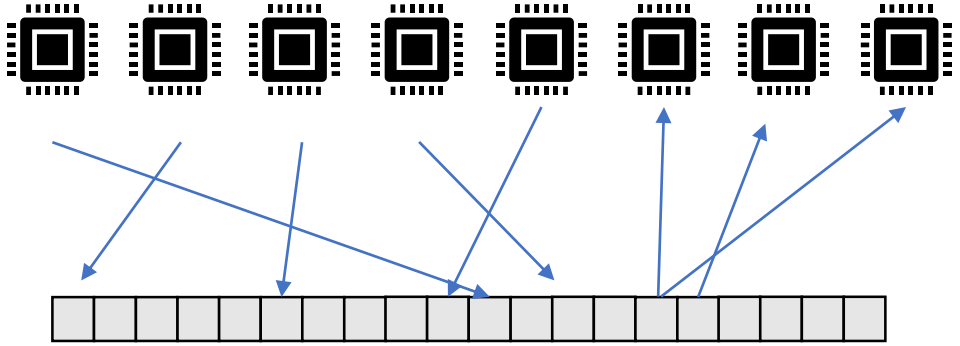
Major Metric:
Round Complexity

MPC

Space $s = N^{\epsilon}$



PRAM



# Rounds

# Parallel steps

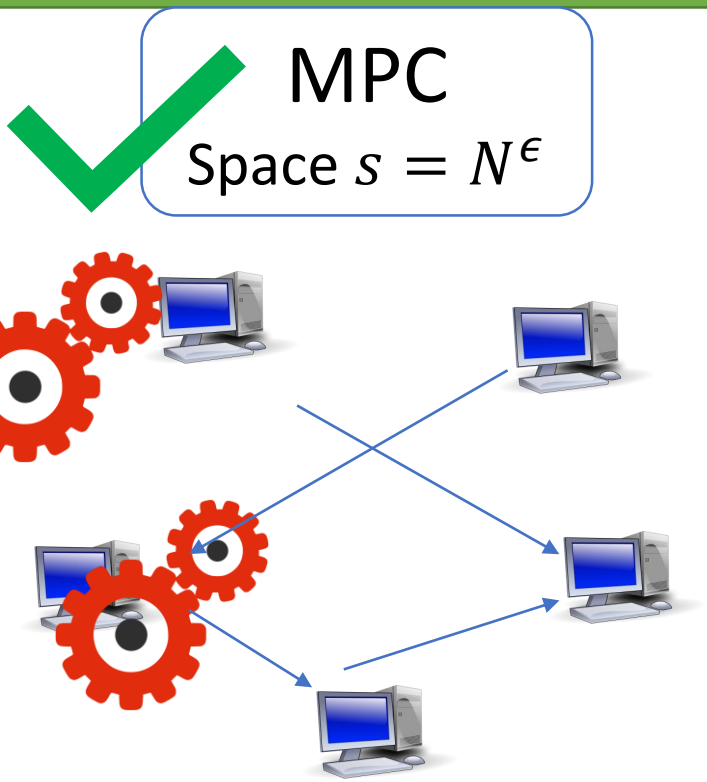
Sort $N$ items

$O(1)$

$\Omega(\log N)$

Same reason motivated MPC (than PRAM) also motivated that …

# Question:
# How to get MPC algo "secure"?
# What is the cost?

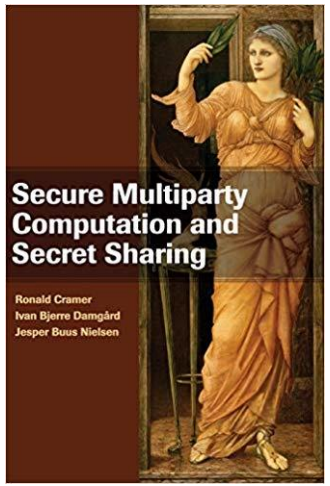


MPC

Space $s = N^{\epsilon}$

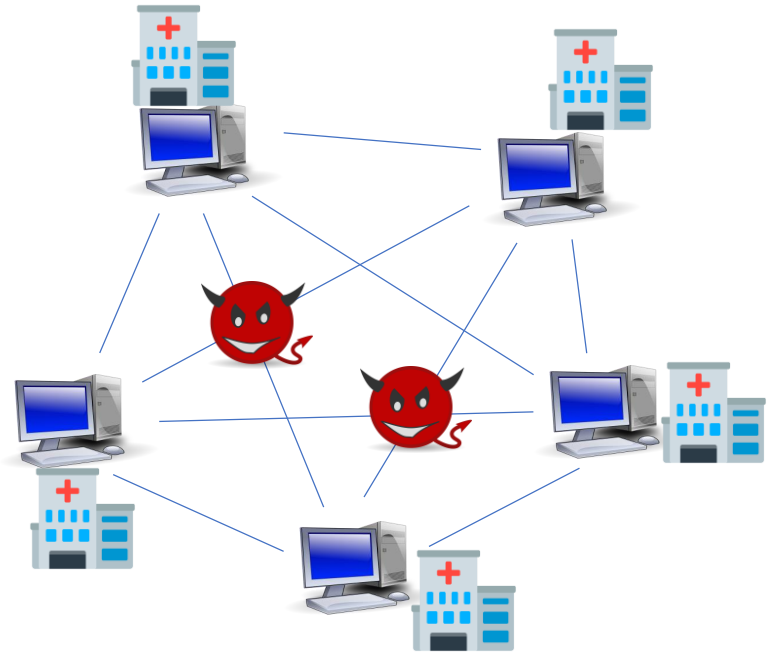
What is "secure" in MPC model?

Many adversarial settings …

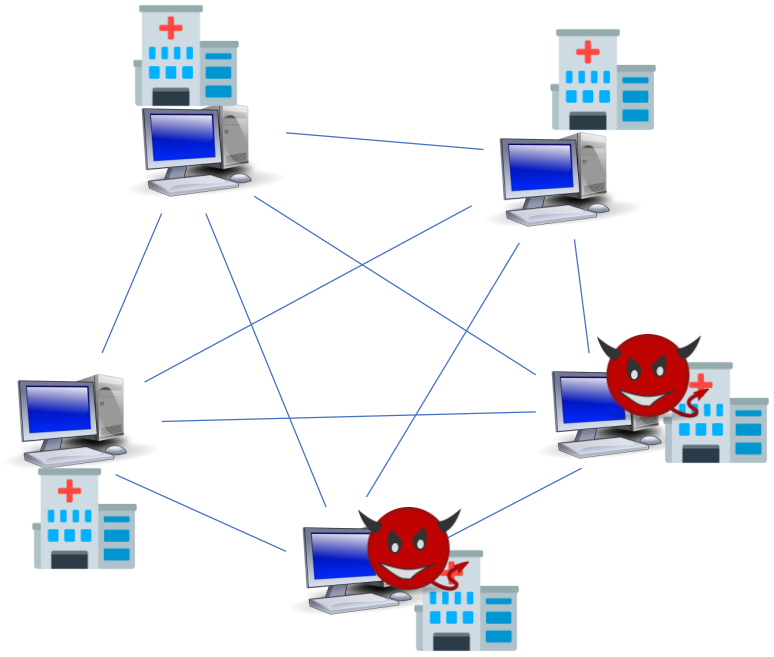



In cryptography, MPC = secure Multiparty Computation

Scenario 1:
Adversary is only eavesdropping,
wants to learn secret input

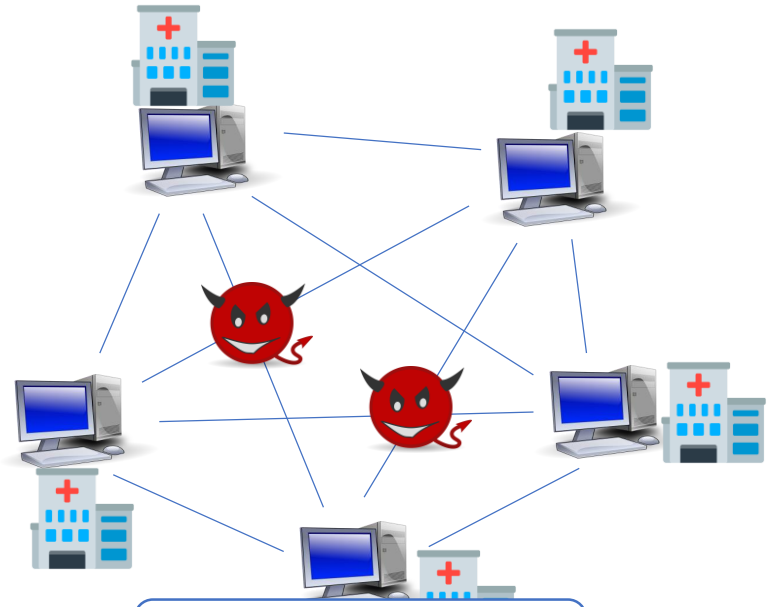


Scenario 2:
Adversary corrupts some machines,
wants secret on others



# Scenario 1: Constant Overhead

Adversary is only eavesdropping, wants to learn secret input



Space $s = N^{\epsilon}$

MPC algo taking space $s$, rounds $R$

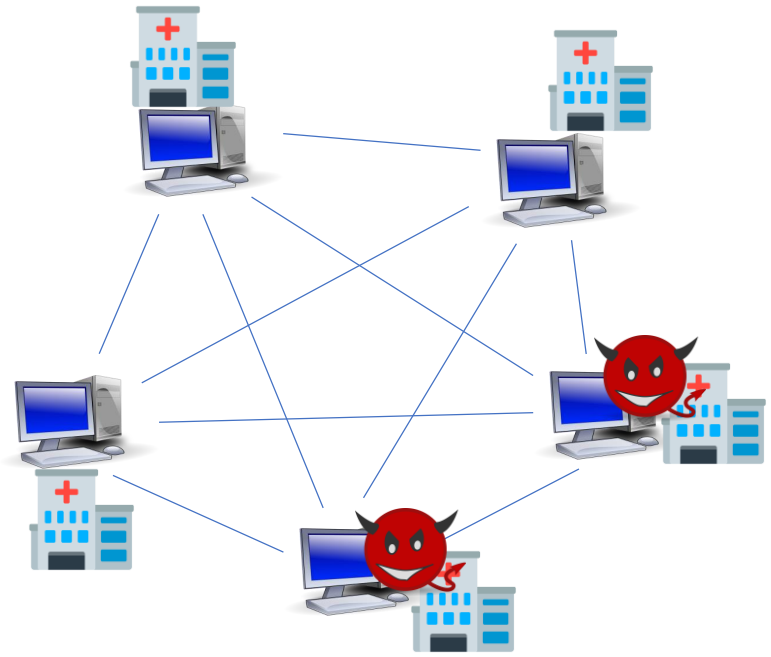
$\Downarrow$

secure MPC algo taking space $O(s)$, rounds $O(R)$

Failure probability in correctness: $\exp\left(-\Omega(\sqrt{s})\right)$

# Scenario 2: Constant in Rounds, Security-Parameter in Space

Adversary corrupts some machines, wants secret on others



MPC algo taking space $s$, rounds $R$

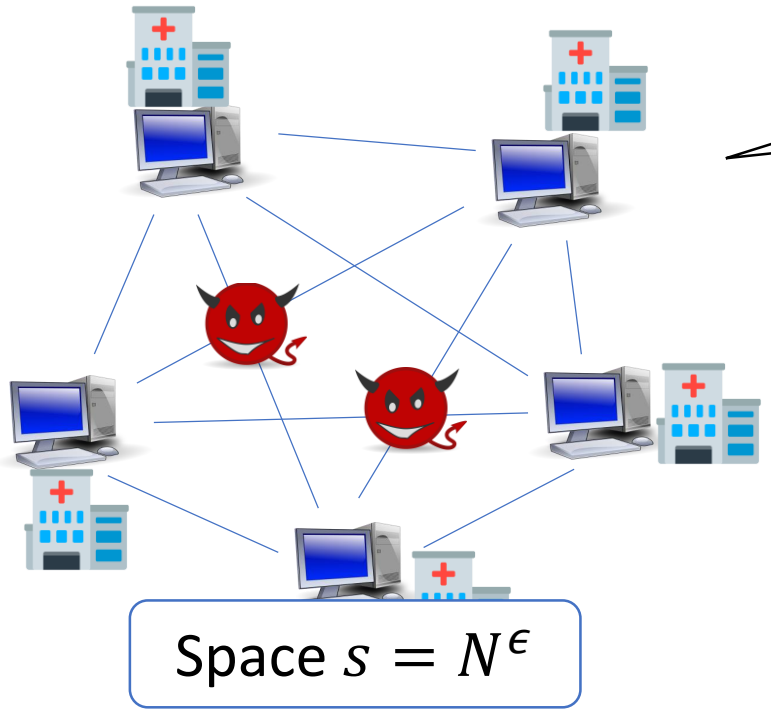
$\Downarrow$

secure MPC algo taking space $O\left(s \cdot poly(\kappa)\right)$, rounds $O(R)$

Assume Learning With Errors (LWE), compact Fully Homomorphic Encryption (FHE), and corrupt machines < 1/3.

Fail probability in correctness: $\exp\left(-\Omega(\sqrt{s})\right)$.

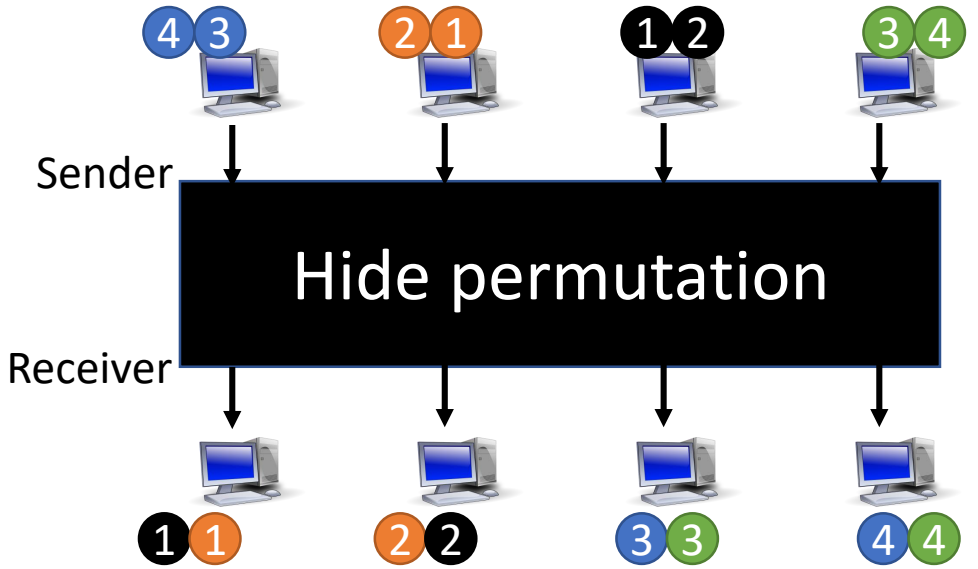
# (Scenario 1) Technique: Oblivious Routing



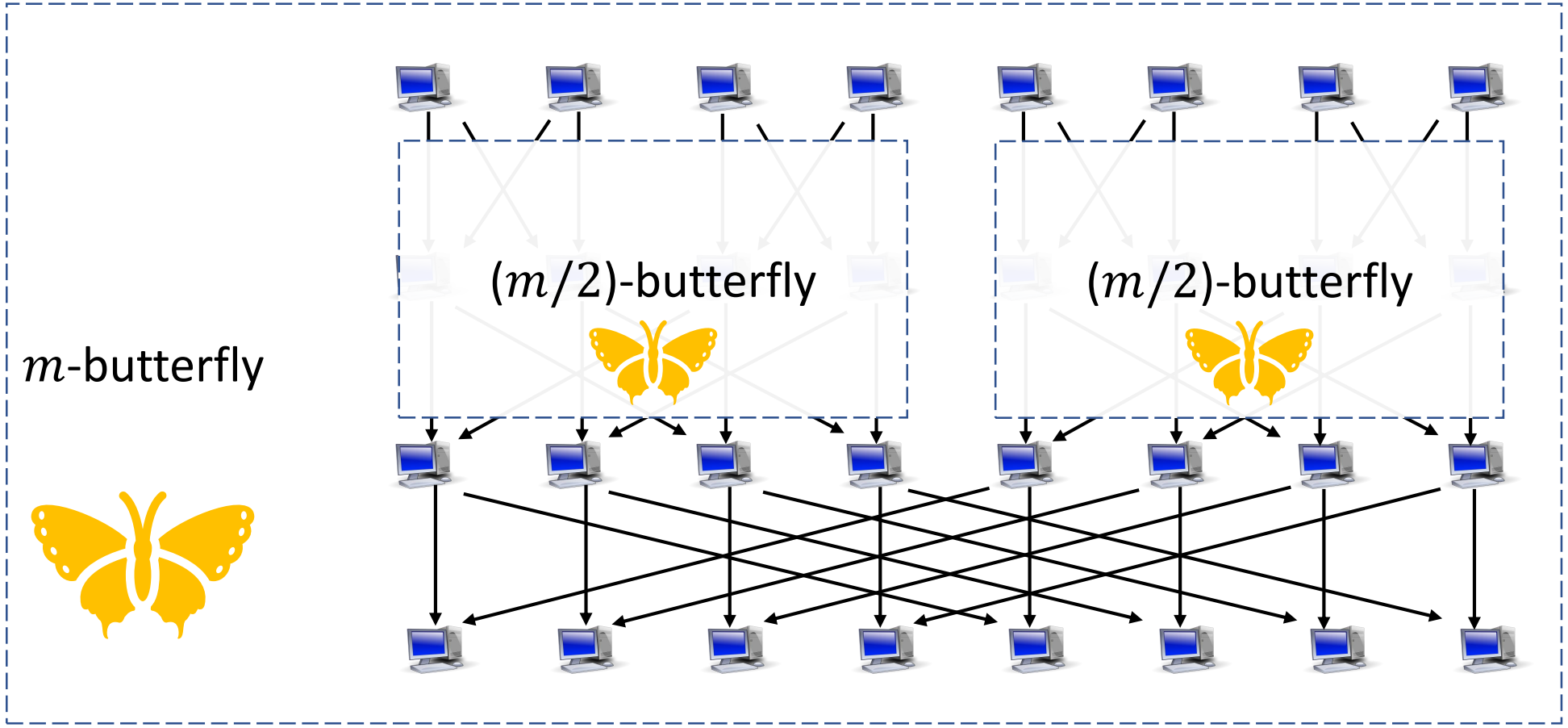
Space $s = N^{\epsilon}$

gets sender-receiver info

Want a **Routing:**

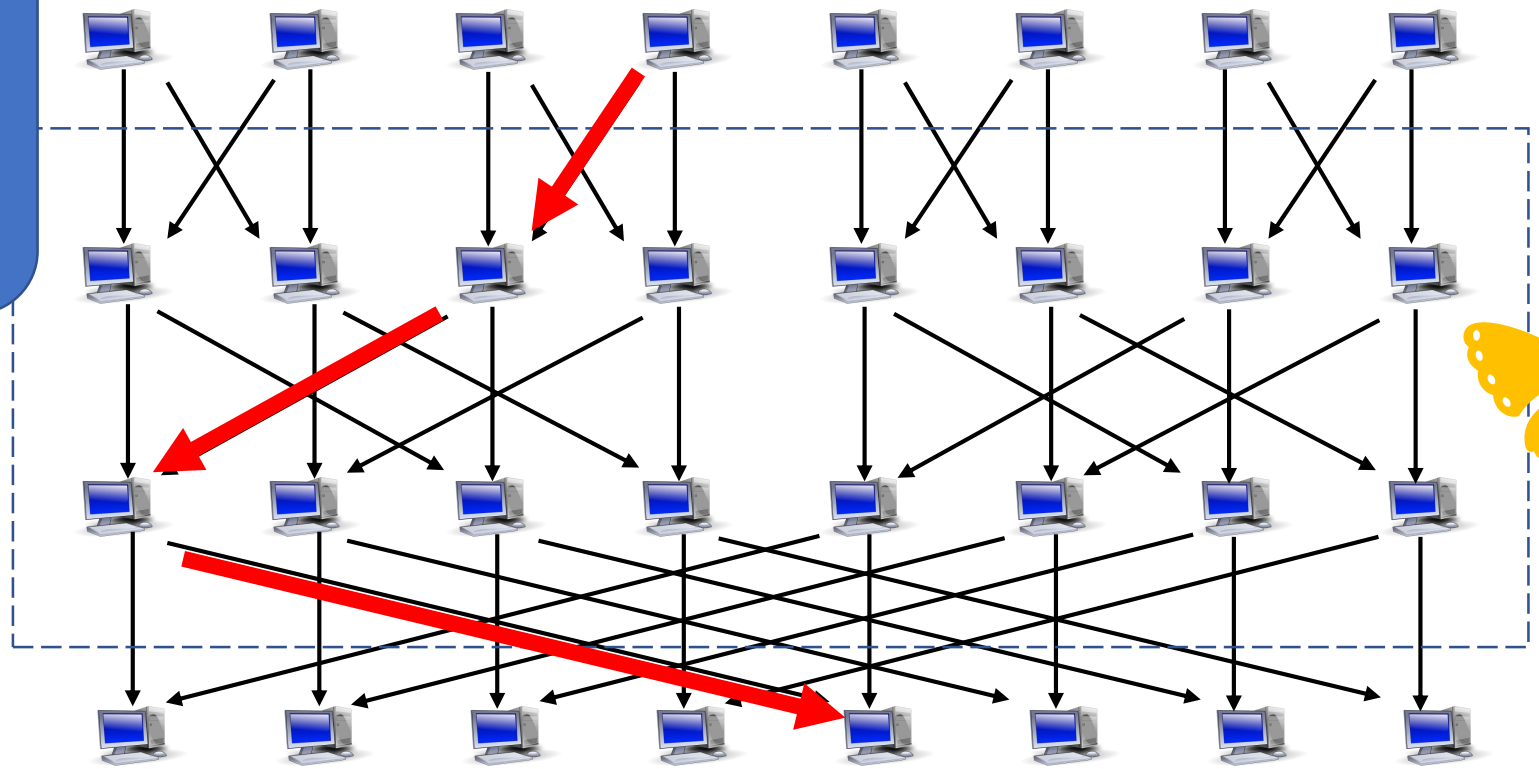


# Butterfly Network (well-known)

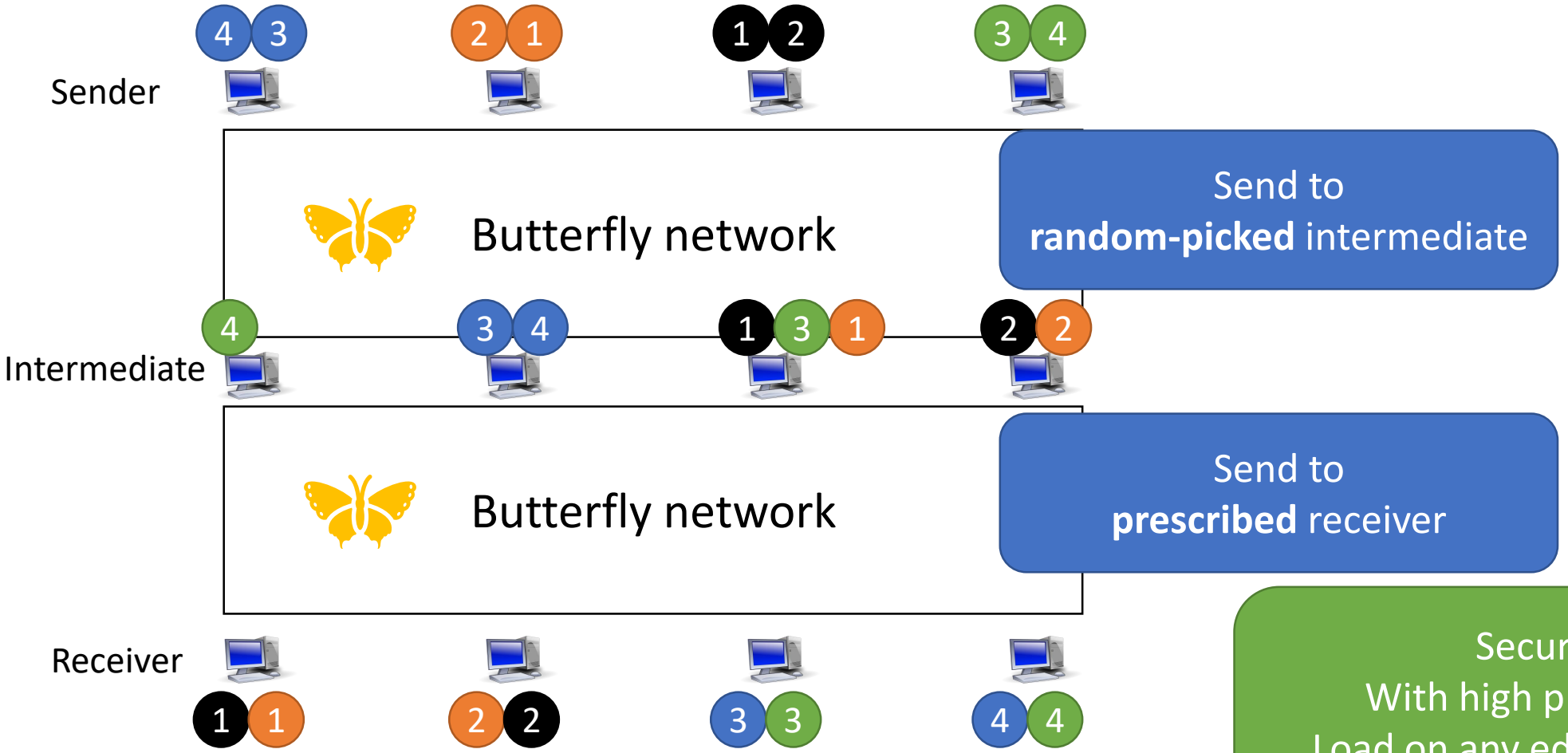


# Butterfly Network (well-known)

Exist a path
for any (sender, receiver),
very easy to find it

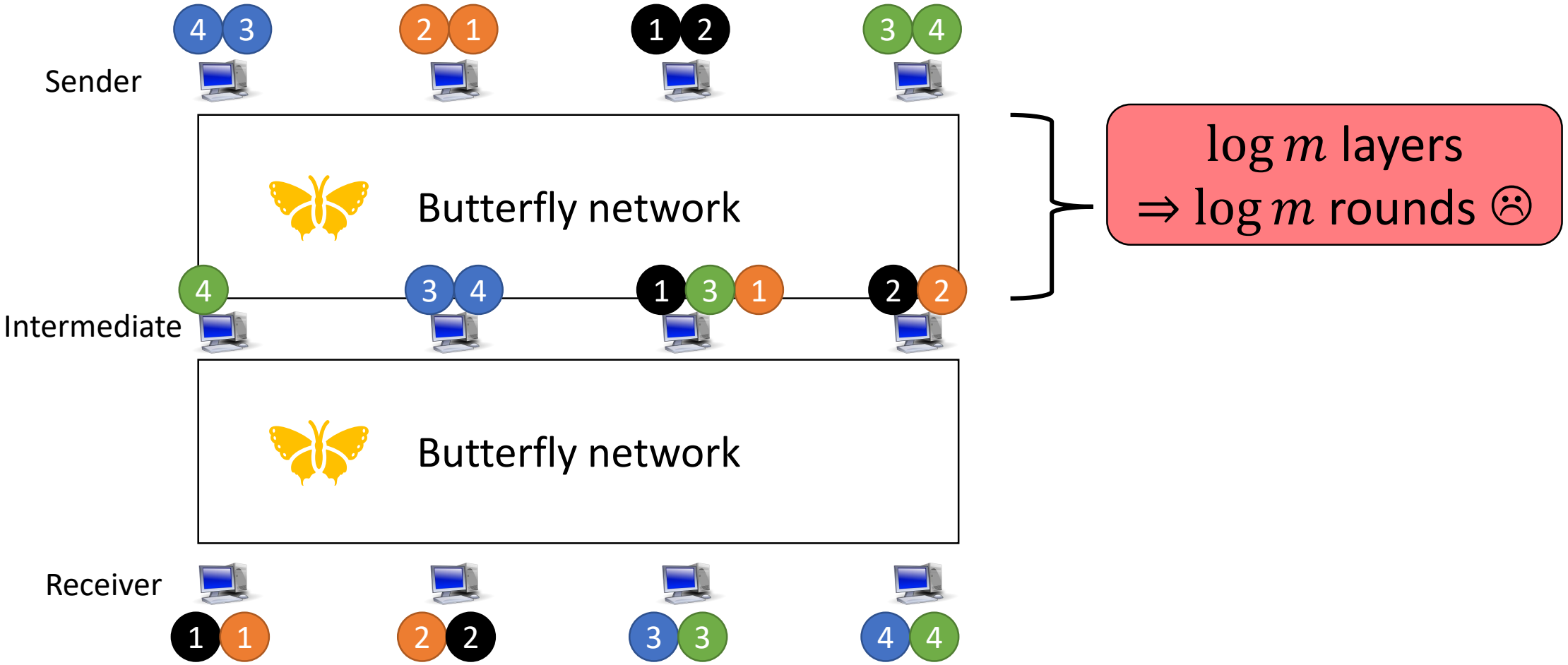


# Routing from Butterfly Network



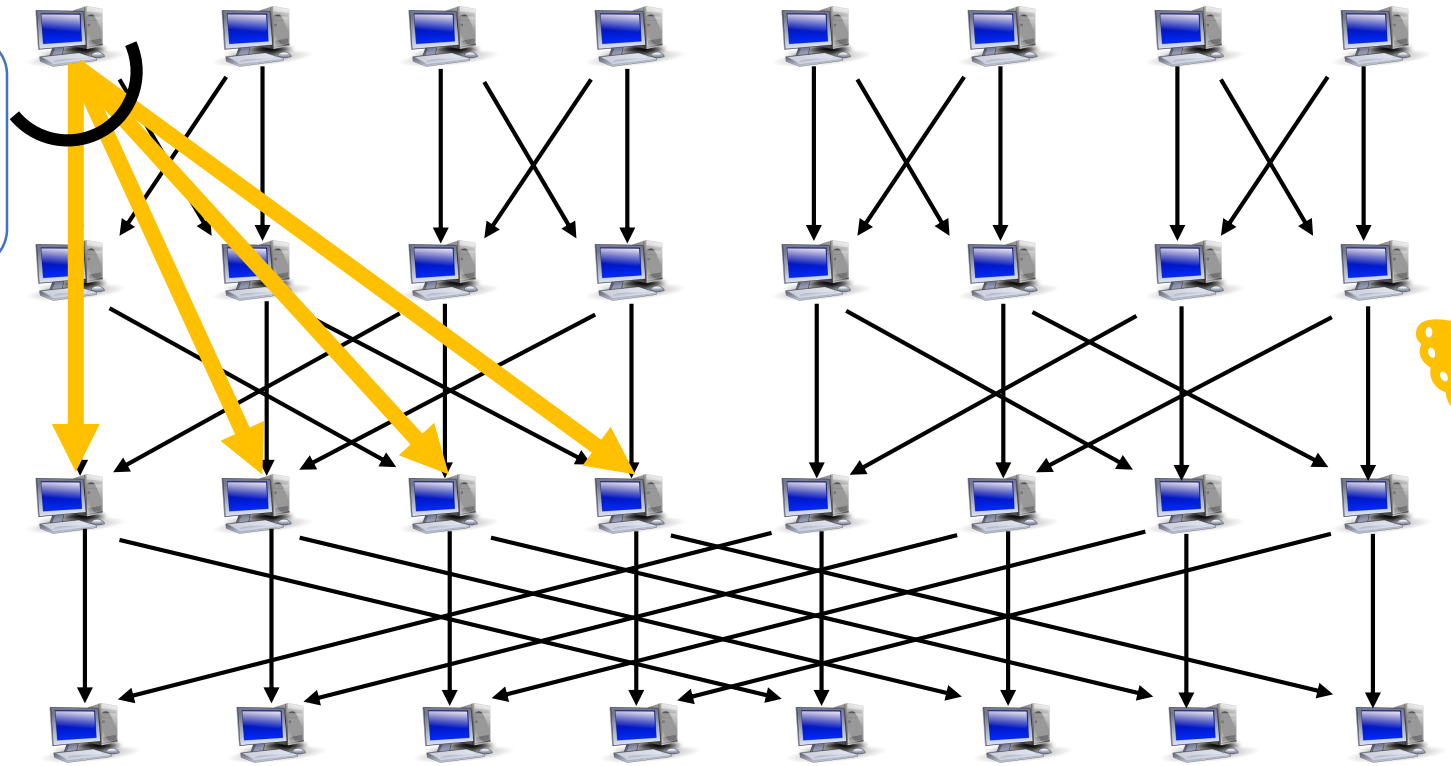

Security:
With high probability,
Load on any edge is at most
const-times than expectation

# Routing from Butterfly Network



# Idea: Degree $s$ Butterfly Network

Use space $s = N^{\epsilon}$ to merge $\log s$ layers



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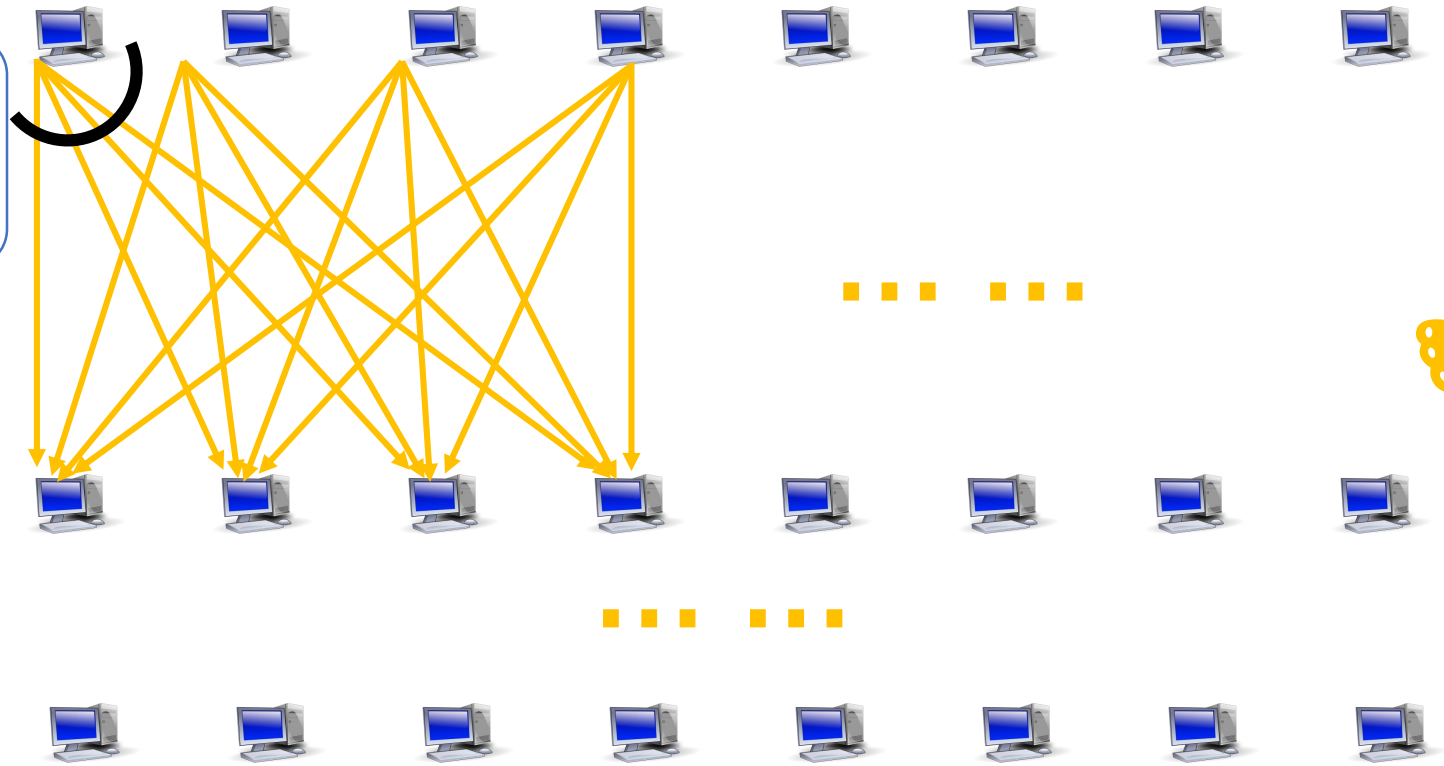

$1/\epsilon$ layers
⇒ const rounds ☺

# Summary

Compile insecure Massively Parallel Computation algo into a secure one

Eavesdropping adversary: const overhead in rounds & space

1/3 corrupt machines:
const overhead in rounds, poly(security para) in space

(Need crypto assumptions)

Thank you!

# Previous result and Discuss

Compare to typical
secure multiparty computation

- Const rounds,
  local space ≈ circuit complexity
- Many rounds, smaller local space

Remove crypto assumptions?

- If we can secure any MPC algo
  using no assumption,
  then we have a statistical SMPC using
  small communication (solve open problem)